Comparison of Fractal Dimension Estimation Methods for Typhoon Wind Speeds

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SUMMARY:
The fractal dimension, a quantitative parameter, represents the characterization of self-similarity in winds. Whereas, the value of fractal dimension of wind speed time histories highly depends on the adopted estimation method. As a result, this paper focuses on the comparison of various estimation methods for the fractal dimension of wind speeds under typhoon climate. Firstly, the structure function method is firstly verified to be more effective and reliable for identifying the fractal dimension of the stochastic Weierstrass Mandelbrot function than the box counting method, variation method and R/S analysis method. Then, the fractal dimensions of wind speed data recorded before and after Typhoon Mangkhut landing are estimated and compared in detail. The variation method could obtain a smaller dimension D than its derivative approach (i.e., box counting method). The R/S analysis method gives the minimum estimate close to the unit and is not applicable to obtain fractal dimensions of the wind speed series. The mean dimension $D \approx 1.7512$ estimated by the structure function method is the closest to the representative value 1.7, which present the best accuracy.

Keywords: Fractal dimension, Field-measured wind speed, Structure function method

1. GENERAL INSTRUCTIONS

The observed activity in many phenomena is a consequence of several invisible layers of movement, and these motions are associated with one level to the next by means of a scaling factor. Processes with this feature were regarded as fractals (Mandelbrot, 1994). The fractal dimension (D) is the basis to investigate simple character for the assembly of these processes (Rubalcaba, 1997).

The fractal dimensional analysis has received increasing attention to study the nature of wind. At present, there exist several algorithms to derive the fractal dimension of a given wind speed time series, including the box counting method (Sarkar and Chaudhuri, 1994), variation method (Dubuc et al., 1987), R/S analysis method (Peters, 1991), structure function method (Ganti and Bhushan, 1995) and so on. There exist sufficient evidences to show that the estimation of the fractal dimension of the wind speed shows significant variability due to the different methods used. Therefore, finding a suitable method to estimate D is an urgent task in practical applications, and it is necessary to conduct a detailed comparative analysis of methods for fractal dimension estimation.

The main objective of this study is to find an appropriate estimation method to estimate the fractal dimension D and provide the comparison results of fractal dimensions by various estimation methods for the measured typhoon wind speed data recorded at different heights.

2. METHODOLOGY

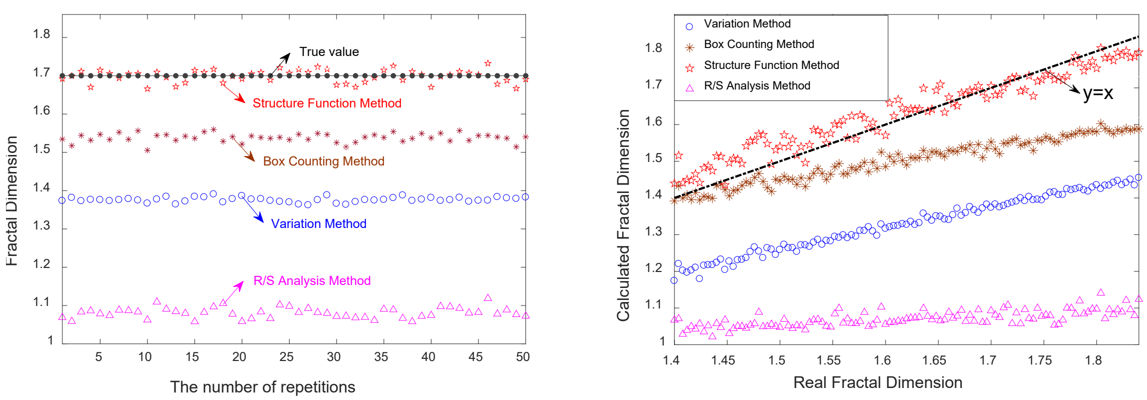
Fractal analysis has become a very useful and powerful tool for studying the underlying dynamics of abundant natural processes, and the fractal dimension D can take non-integer values, ranging from 1 to 2. In this section, four commonly used methods including the box counting method, the variation method, the structure function method and the R/S analysis method will be compared on merit to select the most suitable fractal dimension calculation method for wind speed data.

In order to select the optimal method for estimating the fractal dimension, it is necessary to introduce the Weierstrass Mandelbrot (WM) (Humphrey et al., 1992) fractal function with the known dimension D.

$$R(t) = A \sum^{\infty} \frac{cos(\emptyset_n) - cos(\gamma^n t + \emptyset_n)}{\gamma^{(2-D)n}} \quad (1 < D < 2, \gamma > 1) \qquad (1)$$

where $\emptyset_n$ is taken as a set of random numbers ranging from 0 to 2π, and A is an amplitude parameter. The parameter γ of the WM function determines the density of the spectrum and the relative phase differences between the spectral modes.

In this paper, a fractal dimension of 1.7 was used to generate the time series. Subsequently, the abovementioned estimation methods are employed to calculate the fractal dimension of simulated time series. Considering the existence of the random number $\emptyset_n$ in Eq. (1), different time series may be generated for each simulation. Therefore, 50 iterations of the simulation were implemented with the same control parameters (i.e., $A = 1, D = 1.7$ and $\gamma = 1.08$ in Eq. (1)). Fig. 1(a) exhibits the results of 50 repetitive operations, showing that the random parameter $\emptyset_n$ have little effect on the recognition accuracy of the dimension D. Besides, by taking the value of D from 1.4 to 1.85, Fig. 1(b) shows the corresponding variation of the calculated fractal dimension with the actual value. It can be found that the structure function method is the most applicable and accurate for the identification of fractal dimensions of stochastic WM function.



(a) (b)

Figure 1 Estimated results of fractal dimensions

3. FRACTAL DIMENSION OF TYPHOON WIND SPEEDS

3.1. Data source and description

The data involved in this study were recorded continuously by four three-dimensional (3D) sonic anemometers during super typhoon Mangkhut from September 15 to September 18, 2018, including 526 consecutive 10-min wind speed samples with an accuracy of ± 0.1 m/s. The associated sonic anemometers were installed at the 356 m high meteorological gradient tower in Shenzhen, China (22°38'59''N, 113°53'36''E). The means of longitudinal wind speed data with a fixed period of 10 min before and after Typhoon Mangkhut landing were recorded at length, as shown in Fig. 2. It can be seen that the mean wind speed increases gradually with the increase of the recorded height. Different from the monsoon, it increases and then decreases sharply at each height, rather than remaining constant.

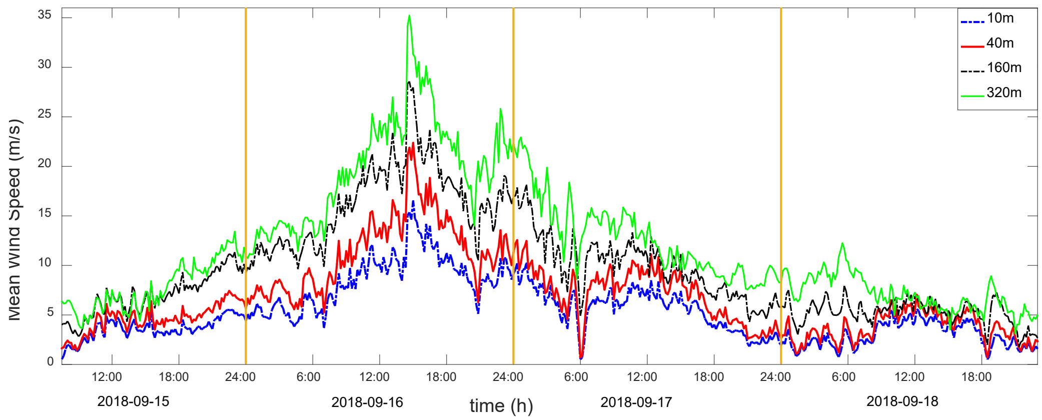


Figure 2 Longitudinal mean wind speeds at each height

3.2. Analysis of fractal dimensions

The analysis in section 2 concluded that the structure function method is more effective than the box counting method, variation method, and R/S analysis method in identifying the fractal dimension of the stochastic WM function. Fig. 3 also shows the variation of the fractal dimension of the longitudinal wind speed samples during the whole measurement period using the four above-mentioned calculation methods. As shown in Fig. 3, the derived fractal dimensions based on the same method became more or less stable throughout the four days of measurements, with the mean dimension $D \approx 1.7512$ estimated by the structure function method being the closest to the representative value 1.7. In the case of the box counting method, its mean estimates for measured heights of 10, 40, 160, and 320 m are 1.5296, 1.5412, 1.5351, and 1.5423. Meanwhile, the variation method could obtain a smaller dimension D than its derivative approach (i.e., box counting method). Fig. 3 also presents that the R/S analysis method could obtain the minimum estimate close to the unit and is not applicable to obtain fractal dimensions of the wind speed series. Instead, the mean Hurst Exponents of the wind speed data $H = 2 - D$ calculated by the R/S analysis method are 0.9526, 0.9658, 0.9694 and 0.9673 at the heights of 10, 40, 160 and 320 m, respectively, which characterizes the strong persistence of wind speeds.

4. CONCLUSIONS

This paper focuses on selecting a suitable estimation method for the fractal dimension of wind speeds. Based on the stochastic WM functions with the known fractal dimensions, the structure function method is proved to be more suitable for identifying the fractal dimension than the box counting method, variation method and R/S analysis method. Field-measured wind data recorded during Typhoon Mangkhut (2018) were further analyzed and discussed in detail by aforementioned four estimation methods. The accuracy of the fractal dimension estimation of the wind speed is significantly affected by the adopted calculation method, where the mean dimension of 1.75 obtained by the structure function method is closer to the representative value of 1.7 than other methods. In addition, although the R/S analysis method is not suggested to compute the fractal dimension, it reveals that obtained reliable Hurst exponents of wind speeds close to 1, which characterizes the strong persistence of wind speeds.

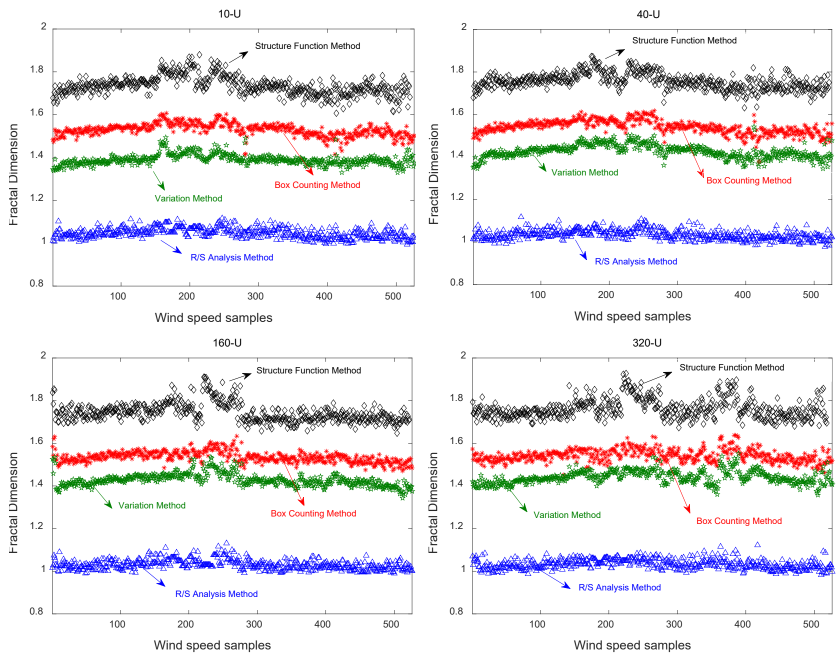


Figure 2 Fractal dimensions of fluctuating wind speed samples at each measured height

Acknowledgments

The work described in this paper was partially supported by National Natural Science Foundation of China (Project No. 52178512), and Zhejiang Provincial Natural Science Foundation (Project No. LQ20E080001).

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